Physics and Physical Measurement

IB 12

Every measured value has uncertainty.

A child swings back and forth on a swing 10 times in 36.27s ± 0.01 s. How long did one swing take?

(36.27 ± 0.01) / 10 = 3.627 s ± 0.001 s

notice that multiple trials reduces uncertainty for a single repetition

Measurements of time are taken as: 14.23 s, 13.91 s, 14.76 s, 15.31 s. 13.84 s, 14.18 s. What value should be reported?

Mean: 14.37 s

Greatest Residual: 0.94

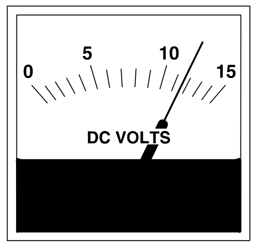
Residuals: 14.37 – 13.84 = 0.53
15.31 – 14.37 = 0.94

Reported Value: 14.37 s ± 0.94 s
= mean ± greatest residual

1. **Measurement:** Record as many significant figures as the calibration of the measuring instrument allows **plus** one estimated digit.

2. **Uncertainty:** Record a reasonable uncertainty estimate that

   a) has one significant digit, and
   b) matches the measurement in place value (decimal place).



Voltage ± uncertainty

V ± ΔV

11.6 V ± 0.2 V

Absolute Uncertainty	**Fractional Uncertainty**	**Percentage Uncertainty**
ΔV	ΔV/V	ΔV/V · 100%
0.2 V	0.2 V / 11.6 V	0.2 V / 11.6 V · 100% = 1.7 %

Calculations with Uncertainties

1. Addition/Subtraction Rule:

When two or more quantities are added or subtracted, the overall uncertainty is equal to the ***sum of the absolute uncertainties***.

Ex. 1: The sides of a rectangle are measured to be (4.4 ± 0.2) cm and (8.5 ± 0.3) cm. Find the perimeter of the rectangle.

4.4 + 8.5 + 4.4 + 8.5= 25.8 cm

0.2 + 0.3 + 0.2 + 0.3 = 1.0 cm

25.8 cm ± 1.0 cm

2. Multiplication/Division Rule:

When two or more quantities are multiplied or divided, the overall uncertainty is equal to the ***sum of the percentage uncertainties***.

Ex. 2: The sides of a rectangle are measured to be (4.4 ± 0.2) cm and (8.5 ± 0.3) cm. Find the area of the rectangle.

4.4 x 8.5
= 37.4 cm^2

0.2/4.4 = 4.55%

0.3/8.5 = 3.53%

Total = 8.08%

37.4 cm^2 ± 8.08%

37.4 cm^2 ± 3.02192 cm^2

37 cm^2 ± 3 cm^2

3. Power Rule:

When the calculation involves raising to a power, ***multiply the percentage uncertainty by the power***.

(Don't forget that $\sqrt{x} = x^{\frac{1}{2}}$)

Ex. 3: The radius of a circle is measured to be 3.5 cm ± 0.2 cm. What is the area of the circle with its uncertainty?

$\textbf{area} = \pi r^2$

$area = \pi(3.5)^2 = 38.48cm^2$

$2\left(\frac{0.2}{3.5}\times 100\%\right) = 2(5.71\%) = 11.4\%$

$area = 38.48cm^2 \pm 11.4\%$

$area = 38.48cm^2 \pm 4.39cm^2$

$area = 38cm^2 \pm 4cm^2$

Exercises

IB 12

1. Five people measure the mass of an object. The results are 0.56 g, 0.58 g, 0.58 g, 0.55 g, 0.59 g. How would you report the measured value for the object's mass?

2. Juan Deroff measured 8 floor tiles to be 2.67 m ±0.03 m long. What is the length of one floor tile?

3. The first part of a trip took 25 ± 3 s, and the second part of the trip took 17 ± 2s.

   a. How long did the whole trip take?

   b. How much longer was the first part of the trip than the second part?

4. A car traveled 600. m ± 12 m in 32 ± 3 s. What was the speed of the car?

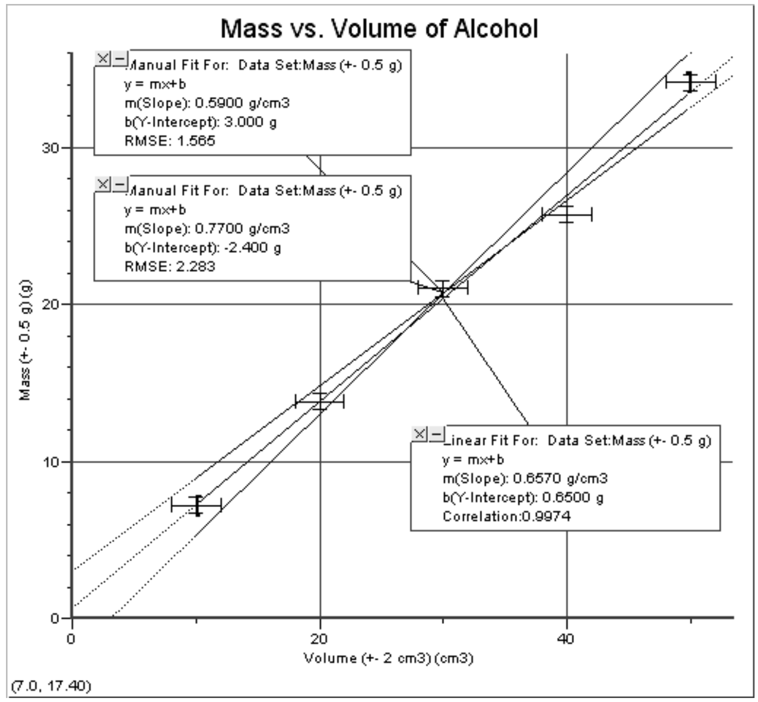
5. The time t it takes an object to fall freely from rest a distance d is given by the formula: where g is the acceleration due to gravity. A ball fell 12.5 m ± 0.3 m. How long did this take?

$$t = \sqrt{\frac{2d}{g}}$$

Analyzing Data Graphically

The masses of different volumes of alcohol were measured and then plotted (using *Graphical Analysis)*. Note there are three lines drawn on the graph – the best-fit line, the line of maximum slope, and the line of minimum slope. The slope and y-intercept of the best-fit line can be used to write the specific equation and the slopes and y-intercepts of the max/min lines can be used to find the uncertainties in the specific equation. The specific equation is then compared to a mathematical model in order to make conclusions.

Data Set	
Volume (+- 2 cm3) (cm3)	Mass (+- 0.5 g) (g)
10	7.2
20	13.8
30	21.0
40	25.7
50	34.1



General Equation: y= mx + b

Specific Equation: M = (0.66 g/cm^3)V + 0.65 g

Uncertainties: slope: 0.66 g/cm^3 ± 0.11 g/cm^3
y-intercept: 0.65 g ± 3.05 g

Mathematical Model: D = M/V so M = DV

Conclusion Paragraph:

1. The purpose of the investigation was to determine the relationship between volume and mass for a sample of alcohol.

2. Our hypothesis was that the relationship is linear. The graph of our data supports our hypothesis since a best-fit line falls within the error bars of each data point.

3. The specific equation of the relationship is M = (0.66 g/cm^3)V + 0.65 g.

4. We believe that enough data points were taken over a wide enough range of values to establish this relationship. This relationship should hold true for very small volumes, although if it becomes too small for us to measure with our present equipment we won't be able to tell, and for very large volumes, unless the mass becomes so large that the liquid will be compressed and change the density.

5. Zero falls within uncertainty range for y-intercept (0.65 g ± 3.05 g) so our results agree with math model and no systematic error is apparent

6. By comparison to the mathematical model we conclude that the slope of the graph represents the density. Therefore the density of the sample is 0.66 g/cm^3 ± 0.11 g/cm^3.

7. The literature value for the density of this type of alcohol is 0.72 g/cm^3. Our results agree with the literature value since the literature value falls within the experimental uncertainty range of 0.66 g/cm^3 ± 0.11 g/cm^3 .

Linearizing Graphs Using Logarithms

This is a special linearizing (straightening) technique that works with general equations that are **power functions.**

Power Function: $y = c \cdot x^n$

Method of straightening: graph log y vs. log x "log-log plot"

Derivation:

$y = cx^n$

Take log of both sides

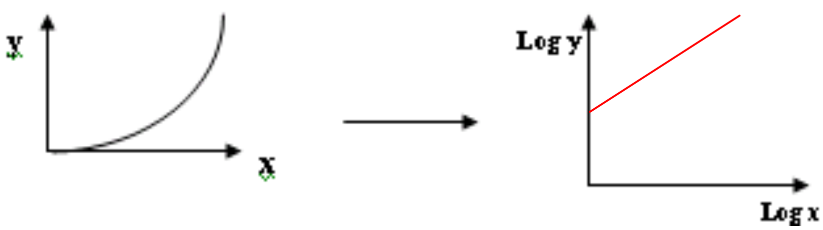
$\text{Log } y = \log (cx^n)$
$\text{Log } y = \log c + \log x^n$
$\text{Log } y = \log c + n \log x$

Compare to $y = mx + b$

slope = n
y-intercept = log c or $c = 10^b$

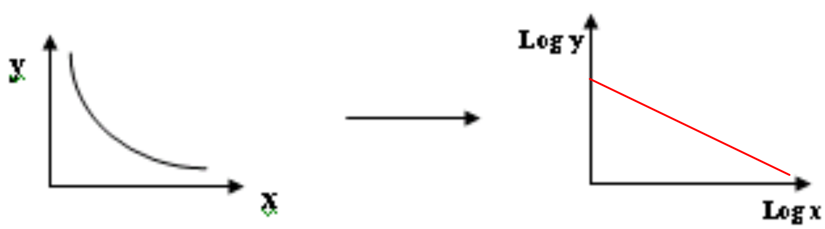
Examples

1. $y = cx^2$



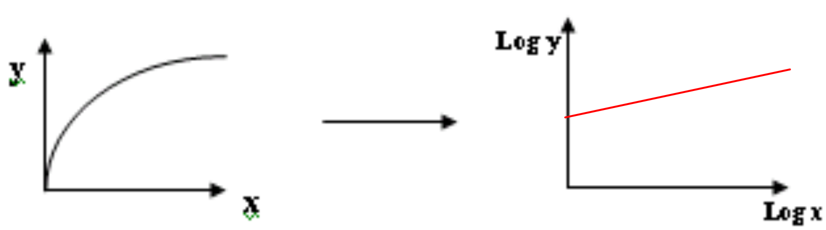
m = 2

2. $y = cx^{-1}$ | $y = kx^{-2}$



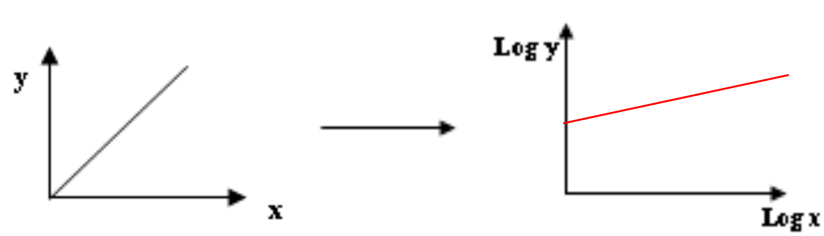
m = -1 or -2

3. $y = kx^{0.5}$



m = 0.5

4. $y = kx^1$



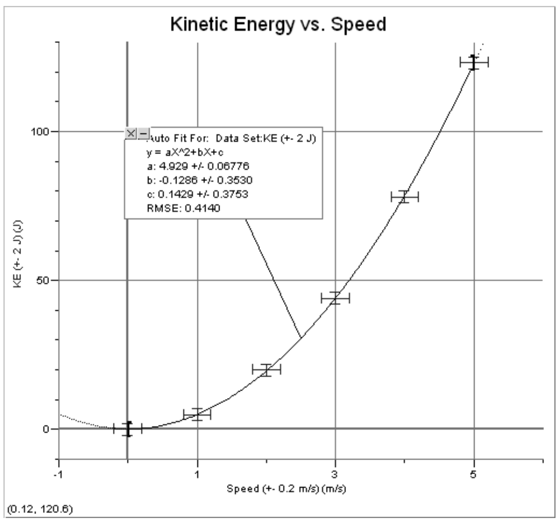
m = 1

Logarithmic Straightening

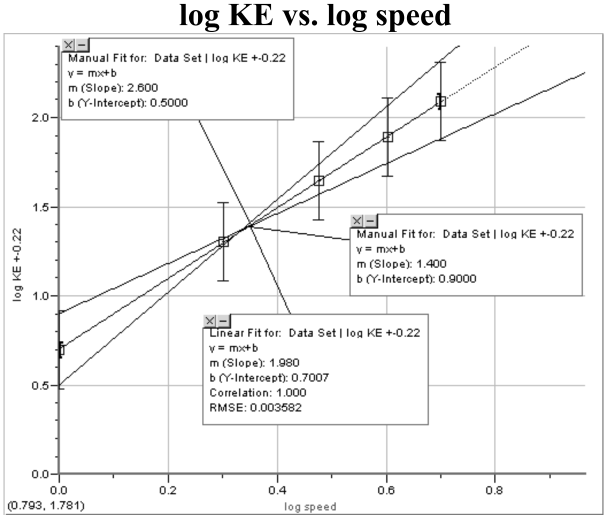
Why use logarithms? use when you're not sure what the type of relationship is – use to check the exponent

Research Question:
What is the relationship between kinetic energy and speed for a uniformly accelerating object?

speed (+- 0.2) (m/s)	kinetic energy (+-2) (J)	log speed	log KE +-0.22
0.0	0		
1.0	5	0.000	0.699
2.0	20	0.301	1.301
3.0	44	0.477	1.643
4.0	78	0.602	1.892
5.0	123	0.699	2.090



log KE vs. log speed



Finding Error Bars for the Straightened Graph:

1. Error bars needed on only one axis – choose whichever axis has the most significant uncertainties.

2. Use the greatest residual for the data point with the highest percent uncertainty as the error bars on all data points.

KE:
log (5-2) = log 3 = .477
log 5 = .699
log (5+2) = log 7 = .845

residuals:
.699 - .477 = .222
.845 - .699 = .146

greatest residual = .222 = .22
use for error bars on log KE axis

Analysis

general equation: $y = cx^n$

slope = 1.980 = n

y-intercept= 0.7007 = log c
so $c = 10^{0.7007} = 5.0$

specific equation: $KE = 5.0\ v^{1.98}$

slope residuals:
1.98 – 1.40 = 0.58
2.60 – 1.98 = 0.62

slope:
1.98 ± 0.62

greatest residual: 0.62

Partial Conclusion:

The purpose of the investigation was to determine the relationship between the kinetic energy and the speed of a uniformly accelerating object. Our hypothesis was that the relationship is quadratic and the graph of our original data supports our hypothesis since a best-fit parabola can be drawn within the error bars of all data points. The data was then linearized using logarithms. Using this graph, the specific equation for the relationship was found to be $KE = 5.0\ v^{1.98}$. Since a value of 2 falls within the uncertainty range for the exponent of 1.98 ± 0.62, the data is consistent with a quadratic relationship between speed and kinetic energy. However, since the uncertainty range for the exponent is so large (30.%), the relationship might not be quadratic but some other power function.

Exercises – Linearizing Data with Logarithms

In each example below, straighten each graph by logarithms. Then, write the specific equation for each relationship. What is the most probable type of relationship in each case?

1.

Time (s) ± 0.2 s	Displacement (m) ± 2 m
0	0
1.0	3
2.0	13
3.0	30
4.0	41
5.0	72

General Equation:
$y = c\,x^n$

Specific equation:
$d = (3.2)t^{1.9}$

Slope:
1.94

Y-intercept:
0.50 = log c
c = 3.2

Type of relationship:
quadratic

2.

Mass (kg) ± 0.1 kg	Acceleration (m/s^2) $\pm 0.1\ m/s^2$
1.1	12.0
2.1	5.9
3.0	4.1
3.8	3.0
5.0	2.5
6.2	2.0

General Equation:
$y = c\,x^n$

Specific equation:
$a = (12)m^{-.99}$

Slope:
-0.99

Y-intercept:
1.083 = log c
c = 12.1

Type of relationship:
inverse

3.

Distance (m) ± 0.1 m	Force (N) ± 0.2 N
1.5	6.7
2.0	3.8
2.4	2.4
3.1	1.7
3.6	1.2
3.9	0.9
4.6	0.7
5.2	0.6

General Equation:
$y = c\,x^n$

Specific equation:
$F = (15)d^{-1.9}$

Slope:
-1.9

Type of relationship:
inverse quadratic

Y-intercept:
1.166 = log c
c = 14.7

Scalars and Vectors

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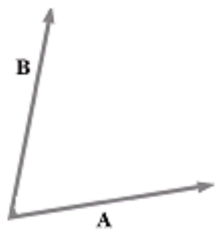
Scalars: quantities that have magnitude only — e.g. - Mass, time, volume, energy, distance, speed

Vectors: quantities that have magnitude and direction — e.g. - Velocity, displacement, acceleration, force, momentum, impulse, magnetic field strength, gravitational field strength, electric field strength

Notation: Bold italic ***F*** or arrow hat $\vec{F}$

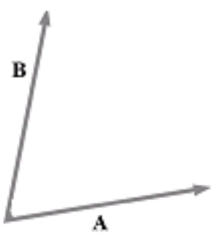
Adding Vectors

Find the sum ***A*** + ***B***

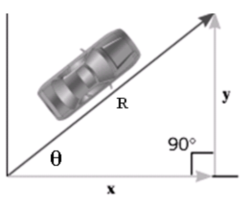


Subtracting Vectors

Find the difference ***A*** - ***B***



Resolving a Vector into its Components



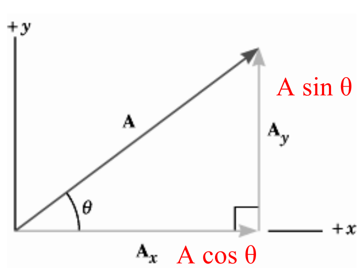
Sin θ = y/r

y = r sin θ

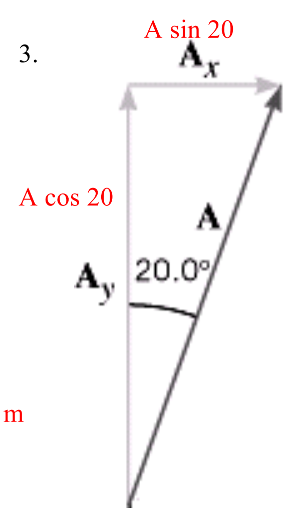
Cos θ = x/r

x = r cos θ

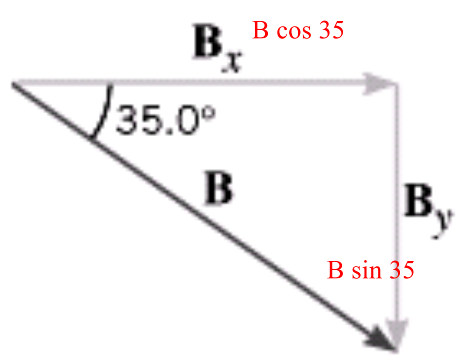
Practice naming components



3.



4.



2.