Diffraction

In previous chapters, we saw the behaviour of light in the *geometrical approximation*, which is when the important phenomena of diffraction and interference are neglected so that we can treat light propagation along straight lines. This chapter deals in detail with the problem of single-slit diffraction and the effect of slit width in the interference pattern.

Objectives

By the end of this chapter you should be able to:

- understand *diffraction* and draw the *diffraction patterns* from a rectangular slit, a sharp edge, a thin tube and a circular aperture;
- appreciate that the *first minimum* in single-slit diffraction past a slit of width b is approximately at an angle $\theta = \frac{\lambda}{b}$;
- draw the *intensity patterns* for a *single slit* of finite width and for *two slits* of negligible width;
- show the effect of *slit width* on the intensity pattern of two slits.

Diffraction

Diffraction, as we have seen earlier, is the spreading of a wave as it goes past an obstacle or through an aperture.

Let us consider a plane wave of wavelength λ propagating toward the right, where an aperture of size b is waiting. What will the wavefronts look like after the wave has gone through the aperture? The answer is not so straightforward. As we will see, the value of the wavelength in relation to the aperture size will be crucial in determining what answer we get. In the first case let us assume that the wavelength is very, very small compared with b (see Figure 7.1).

That part of the wave which is blocked by the screen does not propagate through and only that part which is free to go through does so. If the wave in question is light, this picture says that light goes through the opening, so that if we put a screen beyond the aperture we will see light on an area of the screen identical to the opening and darkness around it. Light travels in straight lines and does not bend as it goes through the aperture. There is no diffraction.

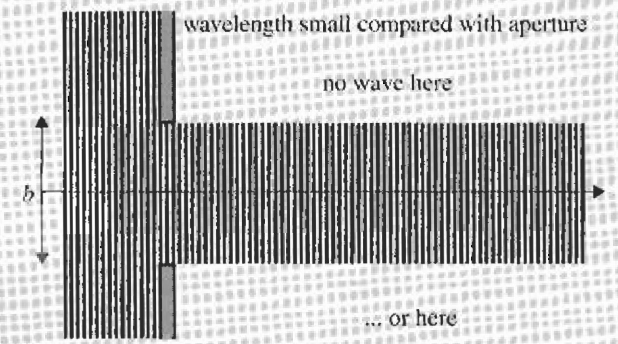

Figure 7.1 When the wavelength is small compared with the size of the opening of the aperture, the amount of diffraction is negligible.

On the other hand, if the wavelength is comparable to or bigger than b, the new wavefronts are curved and the wave manages to go around the edges (see Figure 7.2).

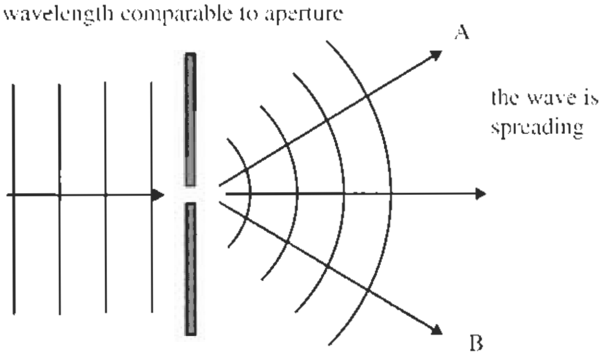

Figure 7.2 When the wavelength is comparable to the opening of the aperture, diffraction takes place.

If we put a screen some distance away from the aperture, we would see light in places where we would not expect any, such as points A and B, for example. This is the phenomenon of diffraction. It takes place whenever a wave whose wavelength is comparable to or bigger than the size of an aperture or an obstacle attempts to move through or past the aperture or obstacle. (Note that here 'comparable to' means that the wavelength can be a few times smaller than the aperture size.)

Diffraction explains how we can hear, but not see, around corners. For example, a person talking in the next room can be heard through the open door because sound diffracts around the opening of the door; the wavelength of sound for speech is roughly the same as the door size. On the other hand, light does not diffract around the door since its wavelength is much smaller than the door size.

Other examples of diffraction are shown in Figures 7.3–7.5.

If the wavelength is much smaller than the obstacle size, no diffraction takes place, as seen in Figure 7.4(a).

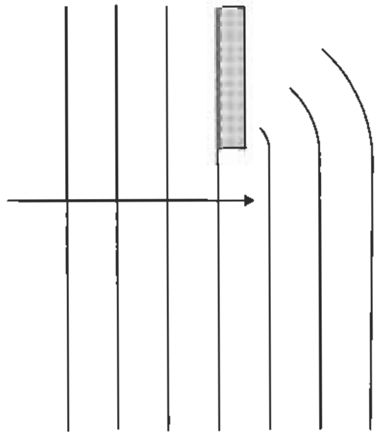

Figure 7.3 Diffraction also takes place when a wave moves past an obstacle.

Diffraction does takes place if the wavelength is comparable to the obstacle size, as seen in Figure 7.4(b).

Figure 7.5 shows interference patterns due to (a) a single slit, (b) two slits, (c) a sharp edge, (d) a circular aperture and (e) a thin cylindrical tube. Notice the wide central maximum in (a) and the secondary maxima in (b). In (c) the pattern consists of bright and dark strips and in (d) of bright and dark rings. In (e) notice the presence of a bright fringe right behind the tube.

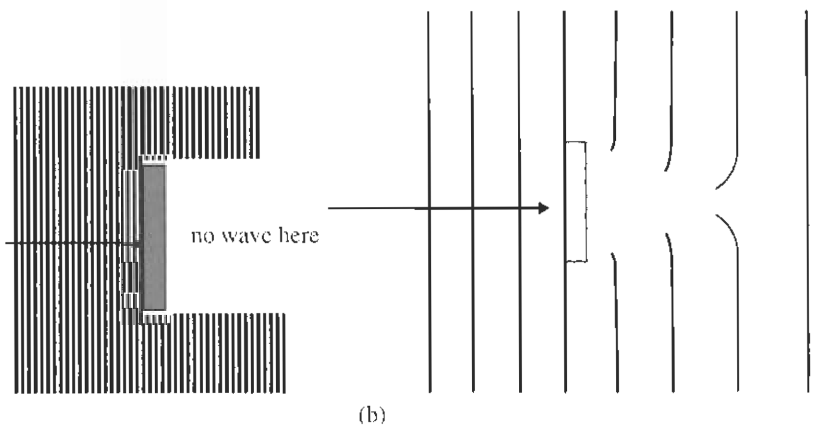

Figure 7.4 (a) If the wavelength is much smaller than the obstacle, no diffraction takes place and a shadow of the object is formed. (b) If the wavelength is comparable to the obstacle size, diffraction takes place and the wave appears far from the object in the region where the shadow was expected.

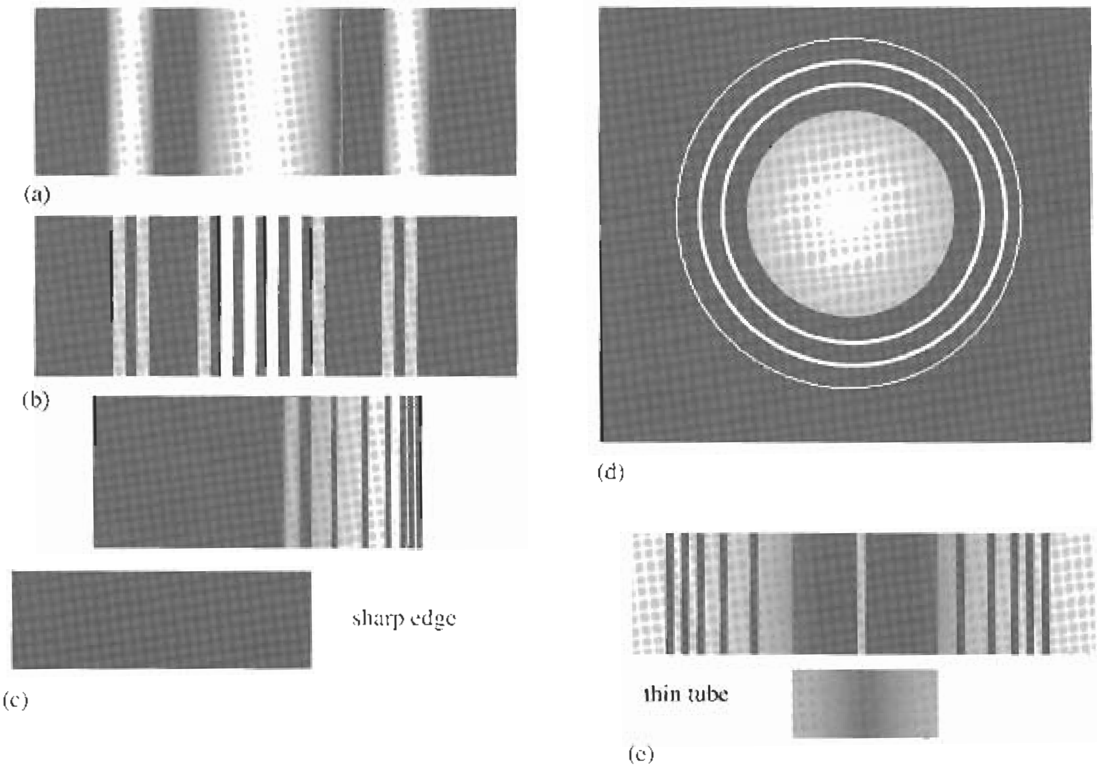

Figure 7.5 (a) Single-rectangular-slit diffraction pattern. (b) Double-slit diffraction pattern. (c) Sharp-edge diffraction pattern. Note that light extends on the inside of the sharp edge. (d) Diffraction pattern from a circular aperture. (e) Diffraction pattern due to a very thin tube with sharp edges. Note that there is light even directly behind the tube.

Single-slit diffraction

When a wave of wavelength λ falls on an aperture whose opening size is b, an important wave phenomenon called diffraction takes place. As we saw earlier:

> **Diffraction is appreciable if the wavelength is of the same order of magnitude as the opening or bigger.**
>
> $\lambda \geq b$
>
> **Diffraction is negligible, however, if the wavelength is much smaller than the opening size.**
>
> $\lambda \ll b$

To investigate this phenomenon we use Huygens' principle (see pp. 234–6) and say that every point on the wavefront that hits the slit will act as a source of secondary coherent radiation. Then what we see at a point P on a screen a large distance away will be the result of the interference of the waves arriving at P from each of the points on the wavefront. Figure 7.6 shows 10 such points labelled A_1, A_2, A_3, A_4, A_5 and B_1, B_2, B_3, B_4 and B_5.

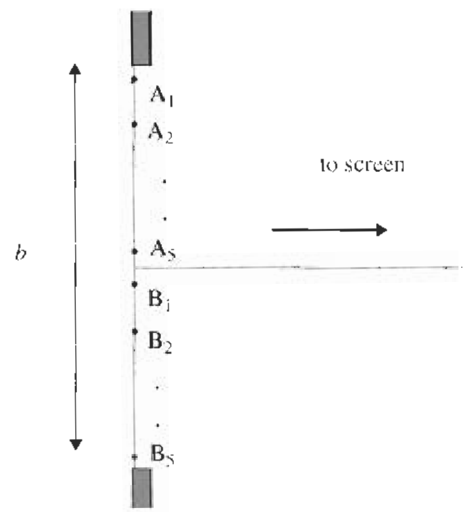

Figure 7.6 In the case of finite slit width each point on the wavefront entering the slit acts as a source of waves according to Huygens' principle and so interference will, in general, result on a screen some distance away.

We choose the Bs in such a way that they are symmetrically placed relative to the As.

All these points are on the same wavefront and therefore are coherent. But, in general, the wave from A_1 will travel a different distance in order to get to P than the wave from B_1 (see Figure 7.7). This path difference will, as in our discussion of interference, result in a phase difference between these two waves at P.

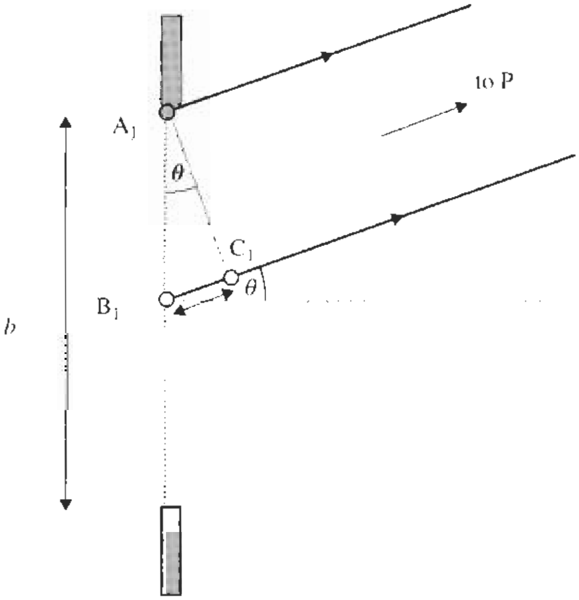

Figure 7.7 Diagram used to calculate the path difference. The path difference equals the distance B_1C_1 . Lines A_1P and B_1P are approximately parallel since P is far away. Thus, triangle $A_1B_1C_1$ is approximately right angled and angle $B_1A_1C_1$ equals θ. The path difference is the length B_1C_1.

If the path difference is half a wavelength, the two waves arrive at P with a 180° phase difference,so the maxima of one wave match the minima of the other. The result is destructive interference, or no wave at P. But remember, we still have to consider the other points, not just A_1 and B_1. What about A_2 and B_2? Triangles $A_1B_1C_1$ and $A_2B_2C_2$ are equal since they are right angled, that is $A_1B_1 = A_2B_2$ and $\angle B_1A_1C_1 = \angle B_2A_2C_2$ (see Figure 7.8). Thus, we see that whatever phase difference exists at P from A_1 and B_1, the same will be true for A_2 and B_2, and so on.

Thus, if we get zero wave at P from the first pair, we will get the same from the second as

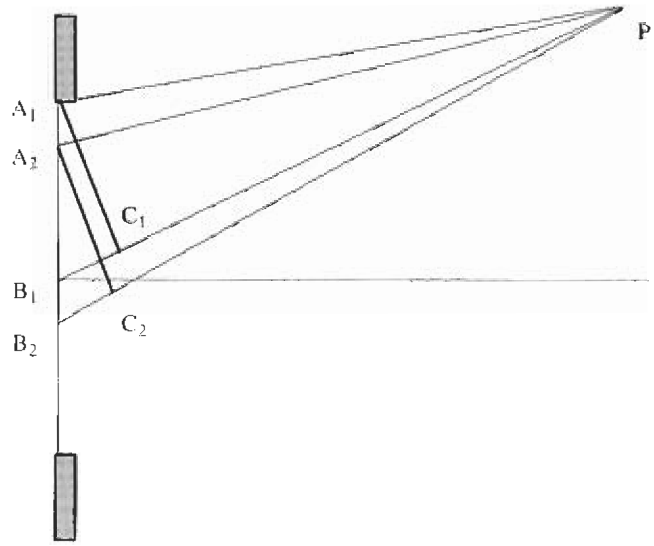

Figure 7.8 Triangles $A_1C_1B_1$ and $A_2C_2B_2$ are equal.

well. Continuing this argument we see that all the points on the wavefront will result in complete destructive interference if the first pair results in destructive interference. To get destructive interference, the path difference must be a half-integral multiple of the wavelength. The path difference between waves arriving at P from A_1 and B_1 is $\frac{b}{2}\sin\theta$ (see Figure 7.6) and so this means that if

$$\frac{b}{2}\sin\theta = \frac{\lambda}{2}$$
$$\Rightarrow b\sin\theta = \lambda$$

we get a minimum at P. If we split the aperture into four equal pieces instead of two and repeat this argument, we will find that the condition for destructive interference is also

$$\frac{b}{4}\sin\theta = \frac{\lambda}{2}$$
$$\Rightarrow b\sin\theta = 2\lambda$$

▶ In general, in interference from a single slit we get *destructive* interference at points P if

$$b\sin\theta = n\lambda \qquad n = 1, 2, 3, \ldots$$

This equation gives the angle θ at which minima are observed on a screen behind the aperture of size b on which light of wavelength λ falls. Since the angle θ is typically small, we may approximate $\sin\theta \approx \theta$ (if the angle is in radians) and so the first minimum is observed at an angle

$$\theta \approx \frac{\lambda}{b}$$

If the slit is circular, then the formula above becomes

$$\theta \approx 1.22\frac{\lambda}{b}$$

In practice, it makes no difference which one is used as both are approximate expressions anyway.

The maxima of the pattern are approximately half-way between minima. This equation is very important in understanding the phenomenon of diffraction so let us take a closer look.

The first minimum ($n = 1$) occurs at $b\sin\theta = \lambda$. If the wavelength is comparable to or bigger than b, appreciable diffraction will take place, as we said earlier. How do we see this from this formula? If $\lambda > b$, then $\sin\theta > 1$ (i.e. θ does not exist). The wave has spread so much around the aperture, the central maximum is so wide, that the first minimum does not exist. (Remember that diffraction is the spreading of the wave around the aperture, not necessarily the existence of interference maxima and minima.)

If now the wavelength is comparable to b, then again appreciable diffraction takes place and a number of minima and the intervening maxima are visible (comparable means that the wavelength can be a bit less than b). If, on the other hand, $\lambda \ll b$, then from $\theta \approx \frac{\lambda}{b}$ it follows that θ is approximately zero. In other words, the wave goes through the aperture along a straight line represented by $\theta = 0$. There is no wave at any point P on the screen for which θ is not zero. This means that the passage of the wave leaves a shadow of the aperture on the screen. There is no spreading of the wave and hence no diffraction, as we expected.

The intensity of light observed on a screen some distance from the slit is shown in Figure 7.9(a) for the case $b = 2\lambda$ and in Figure 7.9(b) for $b = 3\lambda$ (the vertical units are arbitrary).

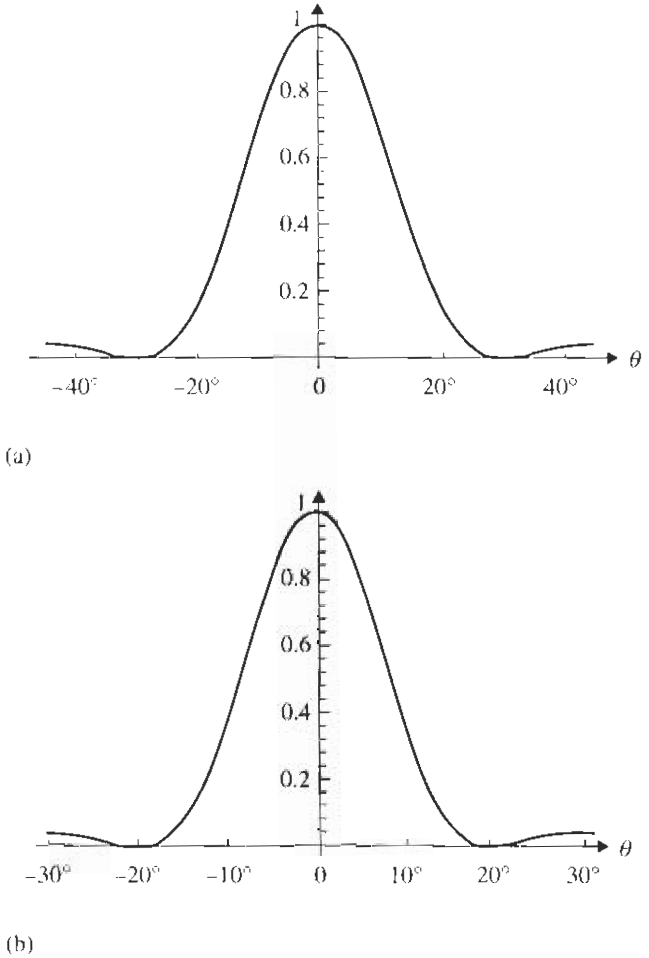

Figure 7.9 The single-slit intensity pattern for (a) a slit of size $b = 2\lambda$, and (b) a slit of size $b = 3\lambda$.

Note that the narrower slit (a) has a *wider* central maximum.

The effect of slit width

At this point it is worth reminding you that in our previous discussion of the Young two-slit interference experiment, we never talked about the size of the slit width, only the separation d of the two slits entered in the formula. This is because we assumed that the slit width was much smaller than the wavelength. As we discussed above, in this case ($\lambda \gg b$) there is no interference pattern from points within the individual slit. The wave just spreads past the slit. The interference pattern we got on the screen in that case was the interference of the two waves after each had spread through each slit. Thus, in this limiting case, if one of the two slits were covered, the interference pattern would disappear.

On the other hand, the case of two slits whose widths cannot be so neglected will result in a more complicated pattern on the screen. This pattern will be the combined effect of (a) the interference pattern from one slit alone and (b) the interference from waves coming from different slits. Let us consider the intensity pattern for two very narrow slits separated by $d = 16\lambda$ shown in Figure 7.10.

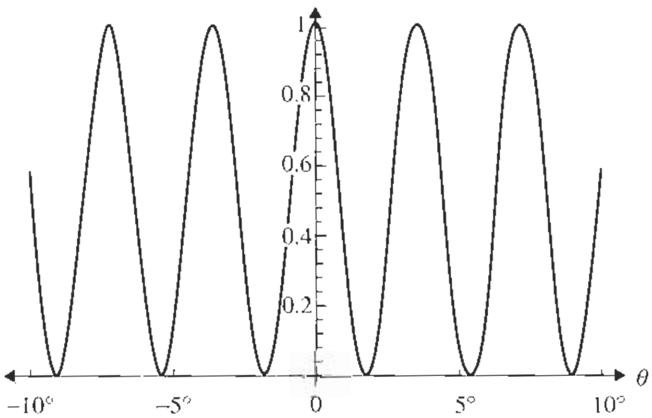

Figure 7.10 The two-slit interference intensity pattern for slits of negligible width separated by $d = 16\lambda$.

The intensity pattern for a single slit of width $b = 3\lambda$ was shown in Figure 7.9(b).

Finally, the intensity pattern for two slits separated by $d = 16\lambda$ as before, but whose width is not negligible, $b = 3\lambda$, is shown in Figure 7.11.

We have shown the single-slit pattern again, which is in fact the envelope curve for the two-slit pattern. The position of the maxima is the same as in the case of the narrow slits but the effect of the slit width is to *modulate* the intensity by the single-slit diffraction pattern.

Missing orders

If the slit width is ignored in a Young-type two-slit interference pattern, we observe a number of equally bright maxima, as in Figure 7.10. If the slit width is not ignored, this intensity pattern will be modulated by the diffraction effects of the slits. It sometimes happens that the first diffraction minimum in the one-slit diffraction pattern coincides with one of the maxima in the two-slit interference pattern. If that happens, the maximum will be reduced to a point of zero

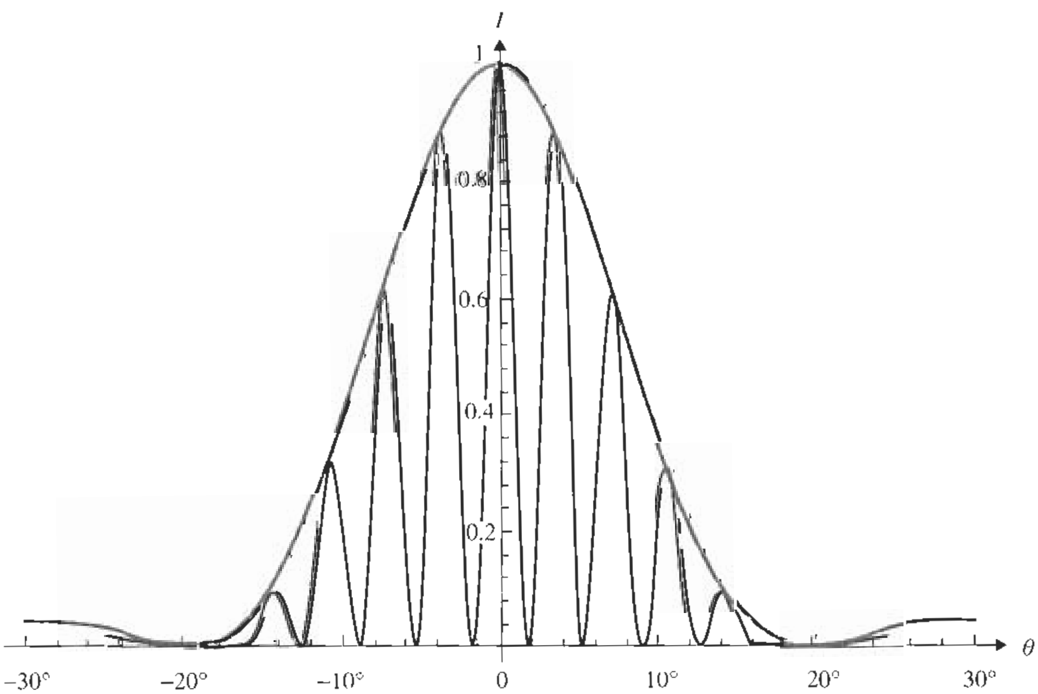

Figure 7.11 The modulated two-slit intensity pattern when the slit width is not negligible. Shown here is the case for $b = 3\lambda$ and $d = 16\lambda$. The heavy curve is the one-slit diffraction curve for a slit width of $b = 3\lambda$.

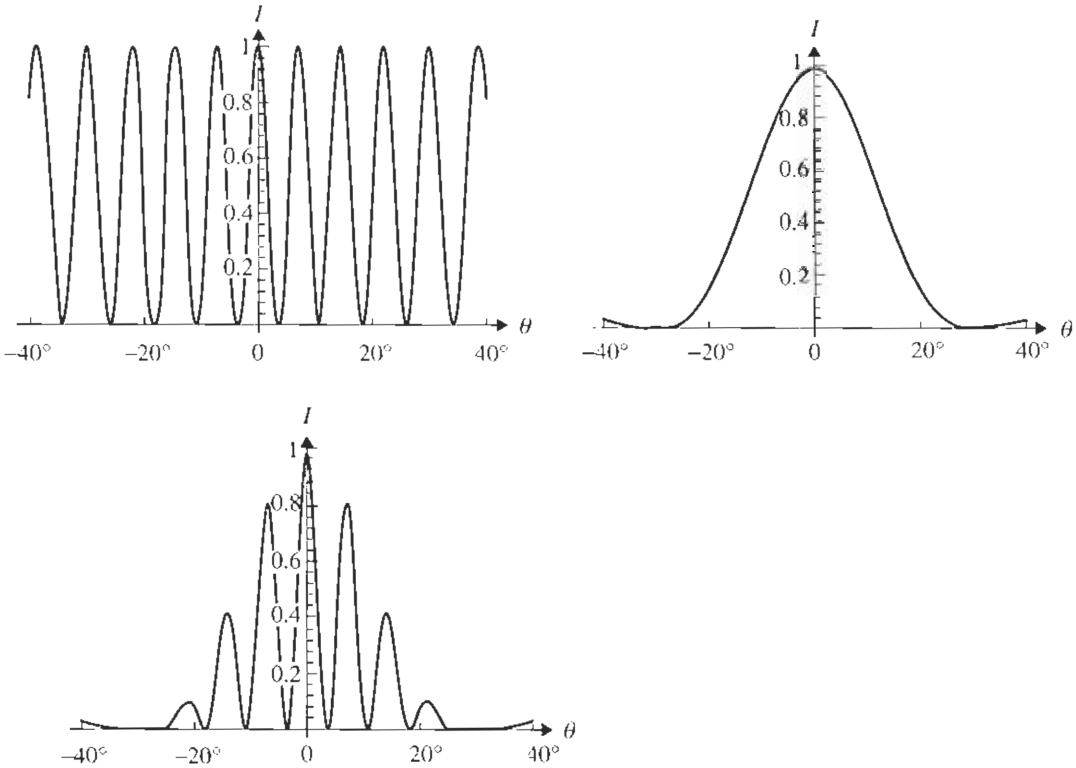

Figure 7.12 The fourth maximum in the two-slit pattern is missing because it coincides with the first diffraction minimum of the one-slit pattern. We can conclude that $d = 4b$.

intensity and we then speak of a missing order. Suppose that the first diffraction minimum occurs at an angle θ. Then $b \sin\theta = 1 \times \lambda$. Suppose that the nth maximum of the two-slit pattern coincides with the first diffraction minimum. Then $d \sin\theta = n\lambda$. Combining the two equations we see that

$$\left.\begin{array}{l} d \sin\theta = n\lambda \\ b \sin\theta = \lambda \end{array}\right. \Rightarrow d = nb$$

that is, the slit separation is n times the slit width where n is the missing order. Figure 7.12 is an example of this where the missing order is $n = 4$.

Questions

1 A single slit of width 1.50 μm is illuminated with light of wavelength 500.0 nm. Find the angular width of the central maximum.

2 Microwaves of wavelength 2.80 cm fall on a slit and the central maximum at a distance of 1.0 m from the slit is found to have a half-width (i.e. distance from middle of central maximum to first minimum) of 0.67 m. Find the width of the slit.

3 The intensity pattern for single-slit diffraction is shown in Figure 7.13. (The vertical units are arbitrary.)

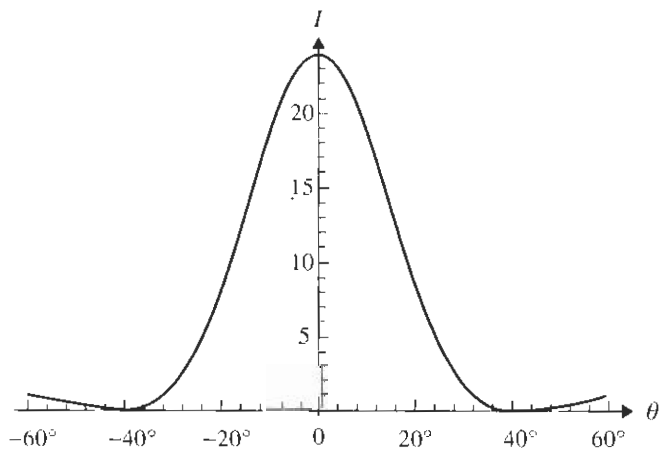

Figure 7.13 For question 3.

(a) Find the width of the slit in terms of the wavelength used.

(b) On a copy of the diagram, draw the intensity pattern for two such slits placed parallel to each other and separated by a distance equal to 10 wavelengths. How many interference maxima fall within the central diffraction maximum?

4 From the information in Figure 7.14, determine the wavelength used to obtain the single-slit diffraction pattern shown. The screen is 0.60 m from the slit and the slit width is 2.30 cm. What kind of wave is most likely being used?

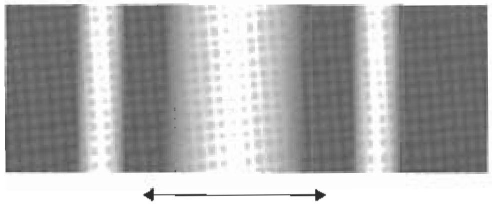

Figure 7.14 For question 4.