Trajectory of a balloon

Introduction

In this investigation the projectile motion of a balloon will be compared to the theoretical model by calculating predicted range and maximum height from the equations of motion. The balloon is projected by stretching it through a cardboard tube (toilet paper roll) and its trajectory measured by analysing video. The aim of the experiment is to find out how close to the theoretical model this motion is and to attempt to explain any deviations.

The balloon launcher

To analyse the motion of a flying balloon it is important that it is projected with the same velocity each time. To launch the balloon it was stretched inside a cardboard tube and suddenly released as illustrated in fig.1.
When the string is pulled elastic potential energy is stored in the rubber of the balloon. The amount of energy is related to the extension of the balloon, if the balloon obeyed Hooke’s law then the elastic PE would equal $\frac{1}{2} kx^2$, rubber does not obey Hooke’s law so the equation for PE is not so simple however the PE stored should be the same if the balloon is stretched by the same amount. To measure the stored PE a force sensor was attached to the string and the Force measured as the extension was increased in steps of 1cm. The results are shown in fig.2

The data from this graph was then used to plot a graph of Force vs extension (fig.3) the area under this graph is equal to the work done in stretching the balloon.

The area gives a value of 0.78J for the elastic PE. Note that the best fit line is a cubic, this was simply chosen as it fits the points almost perfectly. From this graph it is obvious that the spring constant is not constant.

When the string is released the elastic PE will be converted to kinetic energy as the balloon is launched. In an attempt to launch with a constant velocity the string was always pulled so that the balloon knot always reached the end of the tube as in fig.4.
To test if this always gave a constant velocity some preliminary experiments were performed launching the balloon horizontally off the end of the table. This was done by eye, watching where the balloon landed each time and marking the position on the floor with a piece of chalk. The results are shown in Table 1.

| Range of balloon launched from table/cm ± 2cm |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 117             | 117             | 134             | 135             | 142             | 145             | 145             |
| 161             | 175             | 140             | 140             | 145             | 140             | 156             |

Table 1

Due to the method of marking the ball after it had landed the uncertainty in the lengths was estimated to be ±2cm. The range of values is from 117 – 175 cm so the initial velocity does not seem to be constant. It was noticed that releasing the string cleanly was quite difficult, this might have caused the large range of velocities in an attempt to improve this a knot was tied in the end of the string. This knot seemed to be much easier to release cleanly so a second experiment was performed to see if the range was more consistent, the results of launching with a knot are shown in Table 2.

| Range of balloon launched from table with knot/cm ± 2cm |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 155             | 167             | 170             | 178             | 176             | 180             | 180             |
| 180             | 178             | 180             | 178             | 176             | 180             | 178             |

Table 2

Now the range is from 155 – 180. The measurements were made in order; top row followed by bottom row. It is apparent that the launching becomes more consistent with practice.

**Measuring initial velocity**

To measure the launch velocity the motion was recorded with a video camera and analysed with LoggerPro. First attempts were made with a computer webcam but the picture resolution was rather poor so a mobile phone was used instead. The camera was set up on a table next to the motion so that the lens was roughly in the middle of the motion. This was done to reduce any parallax effects that would distort the lengths. To set the scale a 1 m ruler was held in the plane of the motion as in fig.5.
Once the scale has been set the video analysis tools in LoggerPro were used to plot the trajectory of the balloon on the video. Even with the phone camera some of the images were a bit blurry, the balloon often appearing as 3 images. An example of how this was done is shown in fig.6.

Fig.6 marking the position of the blurred ball
Once the trajectory has been marked graphs representing both components of the motion can be plotted. Fig.7 shows the x displacement plotted against time.
Fig. 7 X displacement against time for horizontal launch

From this graph it is clear than the x component of velocity was not constant but decreased over time, this is most likely due to air resistance. The initial velocity of the balloon can be found from the gradient of the first part of the curve. This gives an initial velocity of $3.8 \pm 0.1 \text{ ms}^{-1}$. The uncertainty was generated by LoggerPro based on the spread of data points used to plot the line.

This value can be compared with the value obtained from the conversion of elastic PE to KE. The EPE measured from the F vs x graph was 0.78 J, if this is all converted into KE then $0.78 = \frac{1}{2}mv^2$

$m$ was measured to be 2.2 g so this gives $v = \sqrt{\frac{2 \times 0.78}{0.0022}} = 27 \text{ ms}^{-1}$

This is much more than the velocity measured. It is clear that a lot of the energy is converted to other forms.

**Measuring vertical acceleration**

The graph of Y displacement against time shown in fig. 8
This graph clearly shows the vertical acceleration of the balloon and although the launch was supposed to be horizontal reveals a small vertical initial velocity. If the acceleration is uniform then the equation of the line should be \( s = ut + \frac{1}{2}at^2 \). \( U \) is the initial velocity and \( a \) the acceleration. From the best fit parabola the values can be deduced to be \( u = 0.38 \text{ ms}^{-1} \) and \( a = 7.4 \text{ ms}^{-2} \). This is less than the acceleration of freefall which is due to the effect of air resistance and buoyancy.

The velocity – time graph (fig.7) shows the acceleration to be almost constant, the effect of air resistance would increase as the velocity of the ball increased so air resistance will be ignored on the vertical component of the motion, this would imply that the lower value of acceleration was due to buoyancy. This can be tested by calculation:

The diameter of the balloon = 10 cm
Balloon is roughly spherical so Volume = \( \frac{4}{3} \pi r^3 = 5.2 \times 10^{-4} \text{ m}^3 \)
The buoyant force on the balloon = the weight of air displaced = Volume of balloon x air density x g
Buoyant force = $6.3 \times 10^{-3}$ N
The mass of the balloon = 2.2 g so weight = $21.6 \times 10^{-3}$ N
Resultant downward force on balloon = $15.3 \times 10^{-3}$ N
Acceleration of balloon = $F/m = 15.3 \times 10^{-3}/2.2 \times 10^{-3} = 7.0 \text{ ms}^{-2}$ This is in good agreement with the experimental value.

**Modelling the motion**

Using the initial velocity of the balloon we can use the equations for constant acceleration to model the motion of the balloon in excel. The most simple model would ignore air resistance and buoyancy so that the horizontal motion has velocity and the vertical motion constant acceleration equal to g. Using excel the values of $x$ and $y$ were calculated for a range of times. Fig.10 shows the predicted trajectory of the balloon compared to the measured. Note that the measured trajectory has been adjusted horizontally to take into account the small upwards initial velocity.

![Graph showing experiment compared to freefall prediction](image)

**Fig.10 Experiment compared to freefall**

Correcting for the buoyant force would mean that the downward acceleration would be only $7 \text{ ms}^{-2}$ giving the prediction in fig.11.
This can be seen to give a slightly worse prediction since the time of flight is longer due to the reduced acceleration.

This final prediction was made by reducing the horizontal component of velocity by an amount that was proportional to the previous velocity this models the drag force which is proportional to the velocity.

The spreadsheet used to generate the data is shown below.
Velocity is calculated with the formula \( =A2-C$12\times A2 \) so each subsequent value is less than the previous by an amount dependent on the constant in cell C12. The X displacement is then calculated based on this value. Each step of the motion calculated using the new velocity added on to the previous displacement. The formula used to calculate the second step was \( =A3\times 0.05+C2 \), this is the time interval multiplied by the new velocity plus the previous displacement. The constant was then adjusted until the maximum x displacement was as in the experiment. The value 0.7 gave a reasonably close fit.

Note: Only the freefall prediction is a quadratic best fit, all the others are cubic which gave a much better fit for the range but probably wasn’t the actual equation of the line.

**Conclusion and Evaluation**

The aim of this investigation was to investigate the projectile motion of a small balloon. By measurement and comparing with a theoretical model it was found that the motion of the balloon is affected by both buoyancy and drag. Buoyancy reduces the vertical acceleration to a constant 7 ms\(^{-2}\). If this were the only effect the range of the balloon would be more than if in freefall but experiment shows it is less. The effect of drag compensates for the reduced acceleration so that the balloon does not travel as far as if in freefall.

The way that drag was modelled was based on the knowledge that the drag force is proportional to the velocity so for each 0.05 s step in the model the velocity was reduced by an amount proportional to the previous velocity. This is a simplification of the real situation but gave a very good prediction of the experimental result.

The distance travelled by the balloon is quite short so the vertical velocity didn’t reach a very high value (the final vertical velocity was less than the initial horizontal velocity), had it travelled further the drag force would have to be taken into account in the vertical component as well.

Although only one flight of the ball was mentioned many videos were made however the difficulty in achieving the same initial velocity made it pointless to try and combine multiple runs. Given more time it would be beneficial to analyse more runs comparing each run to the model.

The main weakness in the method was the poor quality of the video, with a better camera with more frames per second and a clearer picture the marking of the ball in LoggerPro would have been much more precise.

One surprise result was how little of the elastic energy was converted into kinetic, as the rubber returns to its original shape some work is done on the gas, maybe this would give a measurable
change in temperature. This loss of energy is in some way not so surprising since the effort required to stretch the rubber is much more than the effort required to make the ball travel the same distance by hitting it.

The original intention was to analyse the motion of the balloon launched at different angles however this has been left for another time. Another interesting extension would be to change size of the balloon to increase the buoyant and drag forces. Keeping the size constant but changing the gas to a mixture of air and Helium would enable the effect of buoyancy to be investigated further.

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